Exercise 1:

$p(\theta) = \frac{1}{2} \quad 0 < \theta < 2$

$\phi = \log\theta$

$p(\phi) = p(\theta)\,|d\theta/d\phi| = \frac{1}{2}\cdot e^{\phi} \quad -\infty < \phi < \log 2$

* A prior on θ <u>induces</u> a prior on $\phi = g(\theta)$.

Exercise 2:

We want $Pr(\tilde{y}_1 < \tilde{y}_2 \mid \vec{y}_1, \vec{y}_2)$.
To compute w/ Monte Carlo sampling we need $p(\tilde{y}_1 \mid \vec{y}_1, \vec{y}_2)$ and
$p(\tilde{y}_2 \mid \vec{y}_1, \vec{y}_2)$

By the independence assumed in our model:
$p(\tilde{y}_i \mid \vec{y}_1, \vec{y}_2) = p(\tilde{y}_i \mid \vec{y}_i)$ for $i \in \{1,2\}$

In general, when $\tilde{y}$ is a new observation from the same population, we call

$p(\tilde{y} \mid y_1, \dots, y_n)$ the "posterior predictive distribution"

$p(\tilde{y} \mid y_1, \dots, y_n) = \int p(\tilde{y} \mid \theta)\, p(\theta \mid y_1, \dots, y_n)\, d\theta$
by rule of marginal probability.

We know how to simulate $p(\tilde{y} \mid \theta)$ (data gen model)
& if we know how to simulate from $p(\theta \mid y_1, \dots, y_n)$
we can proceed w/ Monte Carlo Approximation:

$p(\tilde{y} | y_1, \ldots, y_n) = \int p(\tilde{y} | \theta) \, p(\theta | y_1, \ldots, y_n) \, d\theta$

To approximate the integral above:

1. Sample $\theta^{(i)} \sim p(\theta | y_1, \ldots, y_n)$
2. Sample $\tilde{y}^{(i)} \sim p(\tilde{y} | \theta^{(i)})$