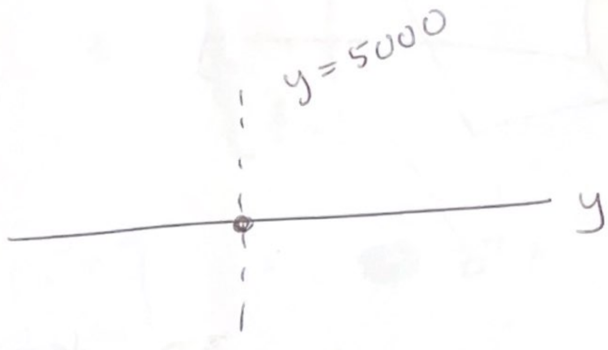
The picture of linear regression.

y : body mass

1D



A linear eqn. in 1D space defines a point. A pt. is a 0-dimensional object.

Let x_1 = flipper length

2D

$y = \beta_0 + \beta_1 x_1$

A linear eqn in 2D space defines a line. A line is a 1-dimensional object

x_2 = bill length

3D

$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2$

A linear eqn in 3D defines a plane. A plane is a 2D object.

In general in D dimensional space a linear eqn defines a <u>D-1</u> -dimensional object and this is called a "hyperplane".

* Be able to identify: sum of squares is equal to a vector inner product.

Ex: $z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$

$$[z_1 \ z_2] \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = z^T z = \sum_{i=1}^{2} z_i^2$$

Ex: $$\underbrace{\sum_{i=1}^{n} (y_i - x_i^T \beta)^2}_{SSR} = (y - x\beta)^T (y - x\beta)$$

Finding $\hat{\beta}_{OLS}$:

Plan take derivative of $\overbrace{(y - x\beta)^T (y - x\beta)}^{\text{sum of square residuals (SSR)}}$ and set equal to 0.

$$\frac{d}{d\beta}\left[\underbrace{y^T y}_{0} - 2\beta^T X^T y + \beta^T X^T X \beta \right]$$

$$= -\underbrace{2 \underset{p\times n}{X^T} \underset{n\times 1}{y}}_{p\times 1} + \underbrace{2 \underset{p\times n}{X^T} \underset{n\times p}{X} \underset{p\times 1}{\beta}}_{p\times 1}$$

← algebra checks out dimension

Now set $= 0$:

$$\cancel{2} X^T X \hat{\beta} = \cancel{2} X^T y$$

left multiply both sides by $(X^T X)^{-1}$:

$$\boxed{\hat{\beta}_{OLS} = (X^T X)^{-1} X^T y}$$

Is it a minimum? Second derivative check:

Yes, $\frac{d}{d\beta}\left[\cancel{-2X^TY} + 2X^TX\beta\right]$

$\cancel{-2X^TY} \to 0$

$= 2X^TX$ ← X^TX symmetric

$\Rightarrow 2X^TX \geq 0$

& if X^TX invertible, then full rank

$\Rightarrow X^TX > 0$