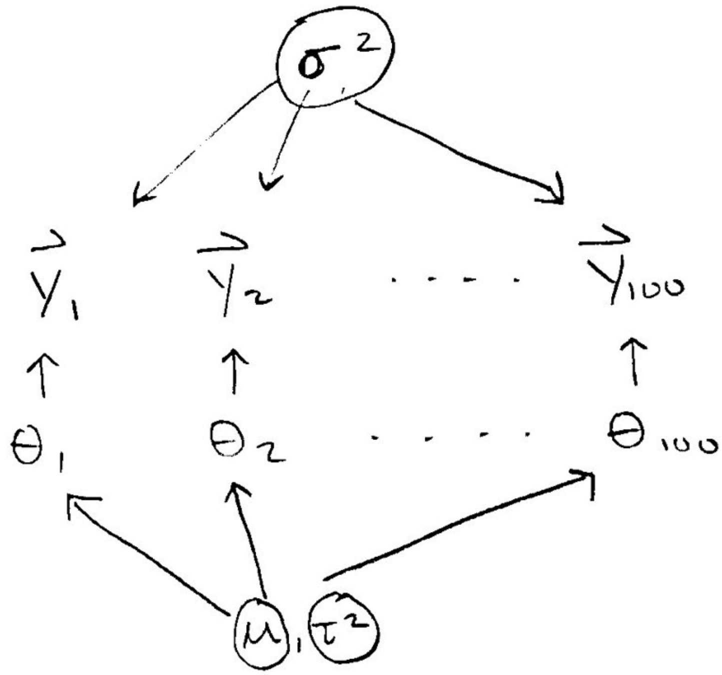
Let y_{ij} = score of student i at school j

$$
\begin{array}{cccc}
y_{1,1} & y_{1,2} & & y_{1,100} \\
y_{2,1} & \vdots & \cdots & \vdots \\
\vdots & & & \vdots \\
y_{n_1,1} & y_{n_2,2} & & y_{n_{100},100}
\end{array}
$$

Let $\vec{y}_j$ be vector of scores at school j.

<u>Hierarchical</u> model:

$$
\begin{tikzcd}
\vec{y}_1 & \vec{y}_2 & \cdots\cdots & \vec{y}_{100} \\
\phi_1 \arrow[u] & \phi_2 \arrow[u] & & \phi_{100} \arrow[u] \\
& \psi \arrow[ul] \arrow[u] \arrow[urr] & &
\end{tikzcd}
$$



$$y_{ij} \mid \theta_j, \sigma^2 \sim N(\theta_j, \sigma^2)$$

$$\theta_j \mid \mu, \tau^2 \sim N(\mu, \tau^2)$$

need priors for: σ^2, μ, τ^2

τ^2: inv-gamma

μ: normal

σ^2: inv-gamma

$$\text{dnorm}(y_{ij}, \theta_j, \sigma^2) = (2\pi\sigma^2)^{-1/2} \exp\left\{-\frac{1}{2\sigma^2}(y_{ij} - \theta_j)^2\right\}$$

To implement a Gibbs sampler requires full cond'l posterior for each unknown.

$p(\theta_j \mid \cdot)$

$p(\sigma^2 \mid \cdot)$

$p(\tau^2 \mid \cdot)$

$p(\mu \mid \cdot)$

* each full cond'l is proportional to the full joint posterior:

$$p(\theta_1, \ldots, \theta_m, \sigma^2, \tau^2, \mu \mid \vec{y}_1, \ldots, \vec{y}_m) \propto$$

$$p(\vec{y}_1, \ldots, \vec{y}_m \mid \theta_1, \ldots, \theta_m, \sigma^2) \cdot p(\theta_1, \ldots, \theta_m \mid \mu, \tau^2)$$

$$\cdot\, p(\sigma^2)\, p(\tau^2)\, p(\mu)$$

Posterior predictive probability

$$p\left(Y^*_{i,51} > Y^*_{i,41} \mid \vec{Y}_1, \ldots, \vec{Y}_m\right)$$

$$= \int p\left(Y^*_{i,51} > Y^*_{i,41} \mid \theta_{41}, \theta_{51}, \sigma^2\right) p\left(\theta_{41}, \theta_{51}, \sigma^2 \mid \vec{y}_1, \ldots, \vec{y}_m\right) d\theta_{41}\, d\theta_{51}\, d\sigma^2$$