1\. Let y_i be # of assists made by a particular player in a particular game.

- Support of y? $y \in \{0, 1, 2, 3, 4, \dots\}$
- $y|\lambda \sim \text{Poisson}(\lambda)$

Let $\lambda = \theta x$

x : # minutes a particular plays in a particular game.

θ : rate of assists per unit time

$$p(y_1, \dots, y_n | \theta) = \prod_{i=1}^{n} p(y_i | \theta)$$

$$= \prod_{i=1}^{n} \frac{(\theta x_i)^{y_i} e^{-\theta x_i}}{y_i!}$$

2\. Prior of unknowns

- What is unknown? answer: θ
- What is its support? answer: $\theta > 0$

$\theta \sim \text{gamma}(a, b)$

Choose $a = 9$, $b = 3$ s.t. $E(\theta) = 9/3 = 3$.

^ terrible prior, can our likelihood overcome it?

3. Posterior

$$p(\theta \mid y_1, \dots, y_n) \propto \underbrace{p(y_1, \dots, y_n \mid \theta)}_{\text{likelihood}} \overbrace{p(\theta)}^{\text{prior}}$$

$$\propto \left(\theta^{\Sigma y_i} e^{-\theta \Sigma x_i}\right) \cdot \left(\theta^{a-1} e^{-b\theta}\right)$$

$$\propto \underbrace{\theta^{\Sigma y_i + a - 1} e^{-\theta(b + \Sigma x_i)}}_{}$$

kernel of gamma(α, β)

$\alpha = \Sigma y_i + a$

$\beta = b + \Sigma x_i$

4. What is $E(\theta \mid y_1, \dots, y_n)$?

"posterior expectation of θ"

$$\frac{\alpha}{\beta} = \frac{\Sigma y_i + a}{b + \Sigma x_i}$$

How is this diff't from prior expectation?

$E\theta = a/b$

Σy_i = total assists

Σx_i = total min / played

prior says $\underline{a}$ assists in $\underline{b}$ minutes.

$var(\theta \mid y_1, \dots, y_n) = \alpha / \beta^2$

4. Poisson as a member of the exp. family

$$p(y \mid \lambda) = \underbrace{\frac{1}{y!}}_{h(y)} \lambda^{y} e^{-\lambda}$$

Now λ^{y} needs to look like $e^{\phi t(y)}$

Let $\boxed{\phi = \log \lambda}$, then $e^{\phi} = \lambda$

$$p(y \mid \phi) = \underbrace{\boxed{\frac{1}{y!}}}_{h(y)} \underbrace{\boxed{e^{\phi y}}}_{e^{\phi t(y)}} \underbrace{\boxed{e^{-e^{\phi}}}}_{c(\phi)}$$

$t(y) = y$

therefore the conj. prior is

$$p(\phi \mid n_0, t_0) = c(\phi)^{n_0} e^{n_0 t_0 \phi}$$

$$= e^{-e^{\phi} \cdot n_0} e^{n_0 t_0 \phi}$$

how to go from $p(\phi \mid n_0, t_0) \rightarrow p(\lambda \mid n_0, t_0)$?

<u>one-line formula</u>:

$$p(\phi \mid n_0, t_0)\, d\phi = p(\lambda \mid n_0, t_0)\, d\lambda$$

$$p(\lambda \mid n_0, t_0) = p(\phi \mid n_0, t_0) \left| \frac{d\phi}{d\lambda} \right| \qquad \frac{d}{d\lambda} \log \lambda = \frac{1}{\lambda}$$

$$\underbrace{e^{-e^{\log\lambda}\cdot n_0}\, e^{n_0 t_0 \cdot \log\lambda}} \cdot \left|\frac{d\phi}{d\lambda}\right|$$

$$= e^{-\lambda n_0}\, \lambda^{n_0 t_0} \cdot \left|\frac{1}{\lambda}\right|$$

$$= \underbrace{e^{-\lambda n_0}\, \lambda^{n_0 t_0 - 1}}_{\text{gamma}(n_0 t_0,\ n_0)}$$